Deciding Memory Safety for Forest Datastructures

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Memory safety is the problem of determining if a heap manipulating program that allocates/frees memory locations and manipulates heap pointers, does not dereference a memory location that is not allocated. Memory safety errors are serious security vulnerabilities that can be exploited systematically to attack systems. In this paper we consider the problem of checking if a program, whose initial allocated heap forms a forest structure (i.e., a disjoint set of trees and lists), is memory safe. While the problem of checking memory safety of programs whose initial heap is a forest structure is undecidable, we identify a class of caching programs for which the problem of checking memory safety is decidable. Our experimental evaluation demonstrates that common library routines that manipulate forest data-structures using a single pass are almost always caching. We show that our decision procedure for such programs is effective in both proving memory safety and in identifying memory safety vulnerabilities.

Additional Key Words and Phrases: memory safety, forest datastructures, verification, decidability

1 INTRODUCTION

The problem of automatic verification is to ascertain whether a program satisfies its assertions on all inputs and all executions. The standard technique for proving programs correct involves writing inductive invariants in terms of loop invariants and pre/post conditions, and proving the resulting verification conditions valid [Floyd 1993; Hoare 1969]. While there has been tremendous progress in identifying decidable fragments for checking validity of verification conditions (Nelson-Oppen combinations of decidable theories realized by efficient SMT solvers [Bradley and Manna 2007]), decidable program verification when annotations are not given has been elusive. Apart from programs over finite domains, very few natural decidable classes are known.

In a recent paper [Mathur et al. 2019], subclasses of uninterpreted programs were identified and shown to have a decidable verification problem. Uninterpreted programs work over arbitrary data domains that give interpretations to the constants, relations, and functions that the program uses, and a program is deemed to be correct only if it satisfies its assertions in all executions when working on all data domains. The authors show that for a class of programs that satisfy a coherence condition, verification is decidable. The decision procedure relies on a streaming congruence closure algorithm realized as automata.

The problem of verifying memory safety

In this paper, our goal is to find classes of programs that manipulate heaps for which memory safety is decidable. Memory safety, in this paper, is defined as follows. A program manipulating a heap

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starts with a set of allocated heap locations, and during its execution dereferences heap pointers, and allocates and frees locations. A program is memory safe if it never dereferences a location that is not in the allocated set. The above definition of memory safety captures the usual categories of memory safety errors such as null-pointer dereferences, use after free, use of uninitialized memory, illegal freeing of memory, etc. [Hicks 2014]. However, in this paper, we do not consider allocation of contiguous blocks of arbitrary size of memory (and hence do not handle arrays and buffer overflows of arrays in languages like C, etc.). Rather, we assume that allocation is done in terms of records of fixed size (like structs in C), and we disallow pointer arithmetic in our programs.

Memory safety errors have attracted a lot of attention because they are serious security vulnerabilities that have been exploited systematically to attack systems [Nagarakatte et al. 2015; Szekeres et al. 2013], and is in fact one of the most common reasons for security bugs [Microsoft 2019]. Memory safety concerns have even led to new programming languages such as Rust that statically assure memory safety (while being efficient). Memory safety vulnerabilities of programs written in C/C++ are still of great concern, and, consequently, identifying fundamental techniques that establish decidability of the problem even for restricted classes of programs is interesting.

There is a rich literature of preventing memory safety errors at runtime by introducing runtime checks by instrumenting code or at compile time (see [Nagarakatte et al. 2015] and references therein, SafeC [Safe-C [n. d.]], CCured [Austin et al. 1994; Condit et al. 2003; Necula et al. 2005, 2002], Cyclone [Jim et al. 2002], SVA [Criswell et al. 2007], etc.). Static checking for memory safety of programs is certainly possible when it is part of language design (for instance, using type systems as in Rust [Matsakis and Klock II 2014]). However, statically verifying memory safety of programs written in languages like C/C++ using abstraction techniques, for example using types or shape analysis [Sagiv et al. 1999], typically result in reporting many false positives.

Our central result in this paper is a technical result that gives efficient decision procedures for verifying memory safety for a subclass of programs, whose initial allocated heap structure is restricted, in a simple imperative programming language.

Let us consider a program (with loops) that we want to prove memory safe, given an initial set of allocated locations. The key insight in this paper is to build decision procedures for memory safety by (a) assuming that the program’s initial allocated heap is a forest datastructure, i.e., disjoint sets of lists and trees, (b) modeling the pointers and their manipulation in the heap precisely using updatable unary functions, and (c) modeling functions and relations on primitive types using uninterpreted functions and relations.

Handling forest datastructures, i.e., disjoint lists and trees, are useful as they are ubiquitous and in fact most common. Note that we require only the initial heaps to be forest datastructures; the program can execute arbitrarily long and create cycles/merges as it manipulates these structures. Second, we model the primitive types and operations on them using uninterpreted functions and relations, similar in spirit to [Mathur et al. 2019] handles all data. The key insight here is that this is a reasonable modeling simplification as programs typically do not rely on the semantics of the primitive data domains in order to ensure memory safety (in fact, we show this also empirically in experiments). However, the salient aspect of our work is that we model the pointers in the heap precisely, without resorting to any abstraction (standard abstractions for heaps like shape analysis involve an abstraction of the heap locations, typically to a finite abstract domain [Sagiv et al. 1999]).

We allow the user to specify the initial allocated set as the (unbounded) set of locations reachable from various locations (pointed to by certain program variables) using particular pointer fields, till a set of locations. The memory safety problem is now to check whether such a program working starting from an arbitrary heap storing a forest datastructure, an arbitrary model for the primitive types, and with the specified allocated set, dereferences only locations that are in the (potentially changing) allocated set of locations on all executions.
The above problem turns out to be undecidable (this is a direct consequence from [Mathur et al. 2019] as even programs that do not manipulate heaps and have simple equality assertions is undecidable). The main result of this paper is that for a class of programs called caching programs, memory safety is decidable.

Technical Challenges
Our starting point is the decidability result of [Mathur et al. 2019] which shows decidability of programs over uninterpreted domains. However, extending this result to our setting is extremely nontrivial.

The primary challenge is to deal with updatable pointers and updatable sets (sets that model the allocated set of locations that keep changing as locations are allocated and freed). First, it is extremely hard to model state using the static model that the work in [Mathur et al. 2019] assumes. The second challenge is handling aliasing. Let \(x\) and \(y\) be two variables that have traversed the heap in some way, and that could point to the same location or point to different locations. In the work of [Mathur et al. 2019], the primary approach is to perform a streaming congruence closure, which computes the minimal set of equalities forced by the execution. However, this would not reveal whether \(x\) and \(y\) point to the same location or not. Now, assume that the program updates a pointer field \(x \cdot p\). We now need to decide whether \(y \cdot p\) has also changed or not, which is not something that is determined. Consequently, the entire approach of [Mathur et al. 2019] fails, and maintaining updated maps and sets become incredibly challenging.

The key idea of our approach is that by restricting to forest datastructures in the initial state, we can keep track of aliasing accurately — when two variables point to locations obtained using different traversals, we know that they cannot alias to each other. Furthermore, by concentrating on caching programs, we can guarantee to keep track of whether traversals for any pair of variables are the same or not.

The ideas above culminate in proving our decidability result that verification of memory safety of caching programs over forest datastructures is decidable and is PSPACE-complete, and is in fact decidable in time that is linear in the size of the program and exponential in the number of variables. We also show that checking whether a given program is caching is decidable in PSPACE. Note that even checking reachability in programs with Boolean domains has this complexity, and hence our algorithms are quite efficient.

Evaluation
We implement a prototype of our automata-based decision procedure. Instead of building the automata and checking emptiness of its intersection with the executions of the program, we build this procedure more efficiently using an approach that constructs the automaton and the intersection on-the-fly, hence not paying the worst case costs upfront. We evaluate our procedure on a class of standard library functions that manipulate forest structures, including linked lists and trees, where various other aspects of the datastructure (such as keys, height, etc.) are modeled using an uninterpreted data domain. These are typically one-pass algorithms on such data-structures that take as input pointers to forest datastrutures, but may manipulate these structures and create non-forest structures during computation.

Though we have stringent requirements that programs must meet in order to be in the decidable class, we show in our experiments that most of these programs pass our requirements. We also show that our tool is able to accurately both verify memory-safety and find memory safety errors in incorrect programs extremely efficiently.

We emphasize that the novelty of our approach is in building decision procedures for verifying memory safety without the aid of human-given loop invariants, and abstracting the data domain
but not the heap domain. In contrast, there are several existing techniques that can prove memory safety when given manually written loop invariants or prove memory safety by abstracting the heap (leading to false positives). Our results hence carve out new ground in memory safety verification and our experiments show that our approach holds promise for wider applicability and scalability.

In summary, this paper makes the following contributions:

- An efficient decision procedure for verifying memory safety for a class of caching programs that dynamically manipulate forest datastructures.
- An efficient decision procedure that determines whether programs are caching.
- An experimental evaluation that shows that (a) common library routines that manipulate forest datastructures using a single pass are often caching, and (b) that the decision procedures for checking whether programs are caching and for checking whether caching programs are memory safe are very effective both for correct and incorrect programs.

The paper is organized as follows. Section 2 defines the class of heap manipulating programs we consider, defining their syntax and semantics, defining the memory safety problem, and proving that it is undecidable in general. We then define the class of structures, namely forest datastructures and the class of caching programs in Section 3. In Section 4, we describe the algorithm for checking memory safety of caching programs on forest datastructures. We describe our implementation and evaluation results in Section 5. We conclude with a discussion of possible future directions in Section 6.

2 PRELIMINARIES

In this section, we define the syntax and semantics of programs that manipulate heaps and other relevant notation useful for presenting the main results of the paper.

2.1 Syntax and Semantics of Heap Manipulating Programs

Programs we consider are those that manipulate heaps. It is convenient to abstract heap structures as consisting of two sorts of distinct elements — a sort Loc of memory locations in the heap, and a sort Data of data values. Field (or map) symbols will model pointers between memory locations, and to data values stored on the heap. Constants and functions over the data domain will be used construct data values to be stored on the heap and in program variables. In this paper, we will not assume any fixed interpretation for either data values or for functions on data values used by programs. In this sense, these programs work over an uninterpreted data domain. Predicates over data values will be modeled by functions capturing the characteristic function of the predicate. We begin by introducing such an abstraction of heaps formally.

Let Loc and Data be the sorts of locations and data respectively. Our vocabulary $\Sigma$ is a tuple of the form $(C_{Loc}, F_{Loc}, C_{Data}, F_{Data}, F_{Loc \rightarrow Data})$, where

- $C_{Loc}$ denote location constant symbols of sort Loc,
- $F_{Loc}$ is a set of unary location function symbols with sort ‘Loc, Loc’ $^1$, that models pointers between heap locations,
- $C_{Data}$ denote data constant symbols of sort Data,
- $F_{Data} = \bigcup_{i \geq 0} F_i$ is such that $F_r$ is a set of data function symbols of arity $r$ of sort ‘Data’, Data’, and

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$^1$We will use the notation $\sigma, r$ to indicate a function whose arguments are from sort $\sigma$ and which returns a value in sort $r$. Thus for example ‘Loc, Loc’ is a function with one argument of sort Loc and which returns an element of sort Loc. On the other hand, ‘Data’, Data’ denotes functions with $r$ arguments each of sort Data and which returns an element of sort Data.
• \( F_{\text{Loc} \rightarrow \text{Data}} \) is a set of unary location function symbols with sort ‘Loc, Data’, modeling pointers to data values stored in heap locations.

For a set \( C \) of constant symbols and a set \( \mathcal{F} \) of function symbols from \( \Sigma \), we denote by \( \text{Terms}(C, \mathcal{F}) \) to denote the set of (well typed) ground terms constructed using the constants in \( C \) and function symbols in \( \mathcal{F} \).

### 2.1.1 Program Syntax

Programs will use a finite set of variables to store information — heap locations and data values — during a computation. Let us fix \( V_{\text{Loc}} = \{u_1, \ldots, u_l\} \) as the set of location variables and \( V_{\text{Data}} = \{v_1, \ldots, v_m\} \) as the set of data variables and let \( V = V_{\text{Loc}} \cup V_{\text{Data}} \) be the set of all variables. In addition, our programs manipulate fields associated with location variables. We will model these fields as functions \( \text{Flds}_{\text{Loc}} = \{p_1, \ldots, p_r\} \) (pointers from locations to locations) and \( \text{Flds}_{\text{Data}} = \{d_1, \ldots, d_s\} \) (pointers from locations to data), and let \( \text{Flds} = \text{Flds}_{\text{Loc}} \cup \text{Flds}_{\text{Data}} \).

Taking \( x, y \in V_{\text{Loc}}, p \in \text{Flds}_{\text{Loc}}, d \in \text{Flds}_{\text{Data}}, a, b \in V_{\text{Data}}, \) and \( e \) to be a tuple of variables in \( V_{\text{Data}} \), the syntax of programs is given by the following grammar.

\[
\langle \text{stmt} \rangle ::= \text{skip} \mid x := y \mid x := y \cdot p \mid y \cdot p := x \mid a := y \cdot d \mid y \cdot d := a \mid \text{alloc}(x) \mid \text{free}(x) \\
| a := b \mid a := f(e) \mid \text{assume}(\langle \text{cond} \rangle) \mid \langle \text{stmt} \rangle ; \langle \text{stmt} \rangle \\
| \text{if} (\langle \text{cond} \rangle) \text{then} \langle \text{stmt} \rangle \text{else} \langle \text{stmt} \rangle \mid \text{while} (\langle \text{cond} \rangle) \langle \text{stmt} \rangle
\]

\[
\langle \text{cond} \rangle ::= x = y \mid a = b \mid \langle \text{cond} \rangle \lor \langle \text{cond} \rangle \mid \neg \langle \text{cond} \rangle
\]

Our programs have well-typed assignments to variables using data values stored in other variables (\( x := y \) and \( a := b \)) or using pointer dereferences from from location variables, either to the data sort (\( a := y \cdot p \)) or to location sort (\( x := y \cdot d \)), or using function computations in the data sort (\( a := f(e) \)). Further, programs can update fields (\( y \cdot d := a \) or \( y \cdot p := x \)), and can dynamically allocate (\( \text{alloc}(x) \)) or deallocate (\( \text{free}(x) \)) memory. In addition, they allow the usual constructs of imperative programming — empty statements (\( \text{skip} \)), conditionals (\( \text{if} - \text{then} - \text{else} \)) and loops (\( \text{while} \)). Conditionals in programs can be Boolean combinations of (well-typed) equality atoms over location or data variables.

### 2.1.2 Program Execution

Execution of programs over \( \Sigma \) and variables \( V \) (given by the \( \langle \text{stmt} \rangle \) grammar) are finite sequences over the alphabet \( \Pi \) given below; as for programs, we use a similar naming convention for variables and fields of different types.

\[
\Pi = \{“x := y”, “x := y \cdot p”, “y \cdot p := x”, “a := y \cdot d”, “y \cdot d := a”, “\text{alloc}(x)”, “\text{free}(x)”, “a := b”, “a := f(e)”, “\text{assume}(x = y)”, “\text{assume}(x \neq y)”, “\text{assume}(a = b)”, “\text{assume}(a \neq b)” \mid x, y \in V_{\text{Loc}}, p \in \text{Flds}_{\text{Loc}}, d \in \text{Flds}_{\text{Data}}, a, b \in V_{\text{Data}}, \text{ and } e \text{ is a tuple of variables in } V_{\text{Data}} \}.
\]

The set of executions of a program \( P \), denoted \( \text{Exec}(P) \) is given by a regular expression inductively defined below. We assume that conditionals are in negation normal forms where “\( \text{assume}(\neg(r = s)) \)”

\[2\text{We use } A \uplus B \text{ to denote the disjoint union of sets } A \text{ and } B.\]
where \( \in C \) translates to “assume\((r \neq s)\)” and “assume\((\neg(r \neq s))\)” translates to “assume\((r = s)\)”.

\[
\begin{align*}
\text{Exec}(\text{skip}) & = \epsilon \\
\text{Exec}(a) & = a \\
\text{Exec}(\text{assume}(c_1 \lor c_2)) & = \text{Exec}(\text{assume}(c_1)) + \text{Exec}(\text{assume}(c_2)) \\
\text{Exec}(\text{assume}(c_1 \land c_2)) & = \text{Exec}(\text{assume}(c_1)) \cdot \text{Exec}(\text{assume}(c_2)) \\
\text{Exec}(\text{if} (c) \text{ then } s_1 \text{ else } s_2) & = \text{Exec}(\text{assume}(c)) \cdot \text{Exec}(s_1) + \text{Exec}(\text{assume}(\neg c)) \cdot \text{Exec}(s_2) \\
\text{Exec}(\text{while} (c) s) & = \langle (\text{Exec}(\text{assume}(c)) \cdot \text{Exec}(s)) \rangle^\ast \\
\text{Exec}(s_1; s_2) & = \text{Exec}(s_1) \cdot \text{Exec}(s_2)
\end{align*}
\]

The set of partial executions of a program \( P \), denoted \( \text{PExec}(P) \), is the set of prefixes of its executions (word in \( \text{Exec}(P) \)).

2.1.3 Program Semantics. The semantics of heap manipulating programs is given in terms of heap structures. A \( \Sigma \)-heap structure is a tuple \( M = (U_{\text{Loc}}, U_{\text{Data}}, I) \), where \( U_{\text{Loc}} \) is a universe of locations, \( U_{\text{Data}} \) is a universe of data. In order to faithfully model dynamic memory allocation, we assume that the set of locations is the disjoint union of a statically allocated set of locations and a countably infinite set of locations that maybe dynamically allocated. That is, we have \( U_{\text{Loc}} = U_{\text{Loc}}^{\text{static}} \uplus U_{\text{Loc}}^{\text{dynamic}} \), where \( U_{\text{Loc}}^{\text{dynamic}} = \{e_0, e_1, \ldots\} \) is an ordered set of distinguished locations indexed by the set of natural numbers \( \mathbb{N} \). The interpretation \( I \) maps every constant \( c \in C_{\text{Loc}} \) to an element from \( U_{\text{Loc}}^{\text{static}} \), every constant in \( C_{\text{Data}} \) to an element from \( U_{\text{Data}} \), every function symbol \( f \in F_{\text{Loc}} \) to an element of \( [U_{\text{Loc}} \rightarrow U_{\text{Loc}}] \), symbol \( f \in F \) of arity \( r \) to an element of \( [(U_{\text{Data}})^r \rightarrow U_{\text{Data}}] \), and, \( f \in F_{\text{Loc}} \rightarrow \text{Data} \) to an element of \( [U_{\text{Loc}} \rightarrow U_{\text{Data}}] \). Further, we assume that the elements in \( U_{\text{Loc}}^{\text{dynamic}} \) cannot be accessed from \( U_{\text{Loc}}^{\text{static}} \), i.e., for every \( f \in F_{\text{Loc}} \), we have \( \forall e \in U_{\text{Loc}}^{\text{static}}, I(f)(e) \in U_{\text{Loc}}^{\text{static}} \).

We assume that corresponding to each program variable \( x \in \mathcal{V} \), there is a distinguished constant \( \bar{x} \in C_{\text{Loc}} \uplus C_{\text{Data}} \) of the appropriate sort denoting the initial value of \( x \). Likewise, each field \( p \in F_{\text{Loc}} \) (resp. \( p \in F_{\text{Data}} \)) is also associated with a unary function \( \bar{p} \in F_{\text{Loc}} \) (resp. \( \bar{p} \in F_{\text{Loc}} \rightarrow \text{Data} \)).

Given an execution \( \sigma \in \Pi^* \) of a program \( P \in \langle \text{stmt} \rangle \) and a \( \Sigma \)-heap structure \( M = (U_{\text{Loc}}, U_{\text{Data}}, I) \), the valuation of the program variables and field pointers at the end of \( \sigma \) are defined in terms of valuation functions \( V_{\text{Loc}} : \Pi^* \times V_{\text{Loc}} \rightarrow U_{\text{Loc}} \), \( V_{\text{Data}} : \Pi^* \times V_{\text{Data}} \rightarrow U_{\text{Data}} \), \( F_{\text{Loc}}V_{\text{Loc}} : \Pi^* \times F_{\text{Loc}}_{\text{Loc}} \rightarrow [U_{\text{Loc}} \rightarrow U_{\text{Loc}}] \) and \( F_{\text{Loc}}V_{\text{Data}} : \Pi^* \times F_{\text{Loc}}_{\text{Data}} \rightarrow [U_{\text{Loc}} \rightarrow U_{\text{Data}}] \), which are presented next. In the following, allocations\((\sigma)\) denotes the number of occurrences of statements of the form “\( \text{alloc}(x) \)”.

\[
\begin{align*}
V_{\text{Loc}}(e, u) & = I(\bar{u}) \\
& = \begin{cases} 
V_{\text{Loc}}(\sigma, y) & \text{if } s = "x := y" \text{ and } u = x \\
\epsilon_i & \text{if } s = "\text{alloc}(x)\”, i = \text{allocations}(\sigma) \text{ and } u = x \\
F_{\text{Loc}}V_{\text{Loc}}(\sigma, p)(V_{\text{Loc}}(\sigma, y)) & \text{if } s = "x := y \cdot p" \text{ and } u = x \\
V_{\text{Loc}}(\sigma, u) & \text{otherwise}
\end{cases} \\
V_{\text{Data}}(e, v) & = I(\bar{v}) \\
& = \begin{cases} 
V_{\text{Data}}(\sigma, b) & \text{if } s = "a := b" \text{ and } v = a \\
I(f)(V_{\text{Data}}(\sigma, c_1), \ldots, V_{\text{Data}}(\sigma, c_r)) & \text{if } s = "a := f(c_1, \ldots, c_r)" \text{ and } v = a \\
F_{\text{Loc}}V_{\text{Data}}(\sigma, d)(V_{\text{Loc}}(\sigma, y)) & \text{if } s = "x := y \cdot d" \text{ and } v = a \\
V_{\text{Data}}(\sigma, v) & \text{otherwise}
\end{cases}
\end{align*}
\]
An execution $\sigma$ is said to be feasible on $M$ if for every prefix of $\sigma$ of the form $\rho \cdot \text{“assume}(x = y)\text{”}$, we have $\text{Val}_\text{sort}(\rho', x) = \text{Val}_\text{sort}(\rho', y)$, and for every prefix of $\sigma$ of the form $\rho \cdot \text{“assume}(x \neq y)\text{”}$, we have $\text{Val}_\text{sort}(\rho', x) \neq \text{Val}_\text{sort}(\rho', y)$, where sort $\in \{\text{Loc, Data}\}$ is the sort of both $x$ and $y$.

2.1.4 Reachability Specification and Memory Safety. Heap manipulating programs are often annotated by a reachability specification that restricts the allowable nodes that can be accessed by a program. A reachability specification is an indexed set of triples $\varphi = (\varphi_k)_{k=1}^n$ where $\varphi_i = (\text{Start}_i, \text{Pointers}_i, \text{Stop}_i)$ is such that $\text{Start}_k \subseteq \text{C}_\text{Loc}, \text{Stop}_k \subseteq \text{C}_\text{Loc}$ and $\text{Pointers}_k \subseteq \text{F}_\text{Loc}$. Each triple $\varphi_i$ denotes a set of locations $\text{Reach}_i$. It is the smallest set such that $\{I(c) \mid c \in \text{Start}_i\} \cup \{I(c) \mid c \in \text{Stop}_i\} \subseteq \text{Reach}_i$, and for every $e \in \text{Reach}_i$ and for every $p \in \text{Pointers}_i$, if $I(p)(e) \notin I(c)$ or $c \in \text{Stop}_i$, then $I(p)(e) \in \text{Reach}_i$. We let $\text{Reach}_\varphi = \bigcup_{i=1}^n \text{Reach}_i$.

Starting with a given reachability specification $\varphi$ on a given heap structure $M$, an execution $\sigma$ defines a set of allocated nodes, which we denote as $\text{Alloc}(\sigma)$ and define as follows.

$$\text{Alloc}(e) = \text{Reach}_\varphi$$

$$\text{Alloc}(\sigma \cdot s) = \begin{cases} 
\text{Alloc}(\sigma) \cup \{\text{Val}_\text{Loc}(\sigma \cdot s, x)\} & \text{if } s = \text{“alloc}(x)\text{”} \\
\text{Alloc}(\sigma) \setminus \{\text{Val}_\text{Loc}(\sigma, x)\} & \text{if } s = \text{“free}(x)\text{”} \\
\text{Alloc}(\sigma) & \text{otherwise}
\end{cases}$$

An execution $\sigma$ is said to violate memory safety over a heap structure $M$ with respect to $\varphi$ if there is a prefix $\rho' = \rho \cdot s$ of $\sigma$ such that $\rho$ is feasible over $M$ and one of the following holds.

1. $s$ is of the form “$w := y \cdot h$” or “$y \cdot h := w$”, $y \in V_\text{Loc}$ and $w$ and $h$ are variables and pointer fields of appropriate sorts, such that $\text{Val}_\text{Loc}(\rho, y) \notin \text{Alloc}(\rho)$.

2. $s$ is of the form $\text{free}(x)$ and $\text{Val}_\text{Loc}(\rho, x) \notin \text{Alloc}(\rho)$.

An execution $\sigma$ is memory safe over $M$ with respect to $\varphi$ if it does not violate memory safety over $M$ with respect to $\varphi$.

With this, we can now define the memory safety verification problem. In the following, we fix our signature $\Sigma$.

**Definition 1** (Memory Safety Verification). The memory safety verification problem asks, given a program $P \in \langle \text{stmt} \rangle$ and a reachability specification $\varphi$, whether for all heap structures $M$, each execution $\sigma \in \text{Exec}(P)$ is memory safe over $M$ with respect to $\varphi$.

We show, unsurprisingly, that checking memory safety is undecidable in general

**Theorem 1** (Undecidability of Memory Safety). The memory safety verification problem is undecidable.
et al. 2019] don’t have heap variables, and do not modify heaps. It was shown (Theorem 11 in [Mathur et al. 2019]) that given an uninterpreted program \( P \), the problem of determining if there is a data domain \( M \) and an execution \( \rho \) of \( P \), such that \( \rho \) is feasible in \( M \) is undecidable. Our result here can be proved by a simple reduction from that problem. Let \( P \) be an uninterpreted program. Consider the reachability specification \( \varphi = (\{D\}x, \{p\}, \{\bar{x}\}) \) such that \( x \in V_{\text{Loc}} \) is new variable. Consider program \( P' = P; y = x \cdot p \). Observe that \( P' \) is memory safe with respect to \( \varphi \) if and only if \( P \) does not have a feasible execution with respect to some data model. □

3 DECIDABLE MEMORY SAFETY VERIFICATION FOR FOREST DATASTRUCTURES

In this section we discuss the formulation of forest datastructures, the class of caching programs and some challenges involving reasoning about the problem of memory safety for heap structures.

3.1 Forest Datastructures

Definition 2 (Forest Datastructures). A heap structure \( M = (U_{\text{Loc}}, U_{\text{Data}}, I) \) over a signature \( \Sigma \) is said to be a forest datastructure with respect to a reachability specification \( \varphi = \{\varphi_k\}_{k=1}^n \) if

1. for every \( 1 \leq i \leq n \), \( \text{Stop}_i = \{\text{stop}_i\} \),
2. for every \( c \in \bigcup_{k=1}^n \text{Stop}_k \), and for every \( f \in \mathcal{F}_{\text{Loc}} \), we have that \( I(f)(I(c)) = I(c) \),
3. for every \( f \in \mathcal{F}_{\text{Loc}} \) and every \( e \in U_{\text{Loc}}^{\text{dynamic}} \), we have \( f(e) = e \), and
4. for every \( t_i \in \text{Terms}(\text{Start}_i, \text{Pointers}_i) \cup \text{Stop}_i \) and \( t_j \in \text{Terms}(\text{Start}_j, \text{Pointers}_j) \cup \text{Stop}_j \) we have, if \( I(t_i) = I(t_j) \), then either \( t_i = t_j \in \text{Start}_i \cap \text{Start}_j \), or \( I(t_i) = I(t_j) = I(\text{stop}_i) = I(\text{stop}_j) \).

Intuitively, a heap structure is a forest datastructure with respect to \( \varphi \), if the subgraph \( G_i \) induced by the set of nodes (excluding \( \text{Stop}_j \)) reachable from \( \text{Start}_i \), using any number of pointers from \( \text{Pointers}_i \) forms a tree, and further, any two subgraphs \( G_i \) and \( G_j \) do not have a node in common (except possibly for the starting locations). Notice that, we do not impose any restrictions on the elements of the data sort of a heap structure.

Theorem 2. The memory safety verification problem for Forest Datastructures is undecidable.

Proof. Follows trivially from Theorem 1 because the reachability specification \( \varphi \) used in the proof of Theorem 1 is such that the \( \text{Reach}_\varphi \) is an empty set. □

3.2 Caching Executions and Programs

In this section, we identify the class of caching programs and executions, for which we show decidability. In order to define these, we will introduce relevant notations.

3.2.1 Terms computed by an execution. An execution \( \sigma \) can be thought of as computing terms in the program variables. For an execution \( \sigma \in \Pi^* \), we define the terms computed by an execution using functions \( \text{Comp} : \Pi^* \times V \rightarrow \text{Terms} \) and \( \text{FldsComp} : \Pi^* \times \text{Flds} \rightarrow [\text{Terms} \rightarrow \text{Terms}] \) defined inductively as follows. We assume the presence of a special constant \( c_{\text{dynamic}} \in C_{\text{Loc}} \) and function \( f_{\text{dynamic}} \in \mathcal{F}_{\text{Loc}} \) which are not used in the programs.
A relation \( \phi \) be an equality relation on terms. The forest equality closure definition does not 'recompute' terms in the sense of terms computed defined above. In this part, we will define the notion of closure equivalence relation such that

\[
R = \{ (t_1, t_2) \mid t_1 \leq_{E} t_2 \} \subseteq \text{Terms} \times \text{Terms}
\]

for every function \( f : \text{sorts} \rightarrow \text{sorts} \), and \( t_i \in \text{Terms} \) of sorts \( w_1, w_2, w_3, \ldots, w_r, w_r, w_r, \) we have

\[
\bigwedge_{i=1}^{r} (t_i, t_i') \in \equiv_{E} \implies (f(t_1, \ldots, t_r), f(t_1', \ldots, t_r')) \in \equiv_{E}
\]

A relation \( R \) is said to be a congruence relation if \( R = \equiv_{E} \).

**Definition 3 (Forest Equality Closure).** Let \( \varphi = \{ \varphi_i \}_{i=1}^{n} \) be a reachability specification, with \( \varphi_i = (\text{Start}_i, \text{Pointers}_i, \text{Stop}_i) \). Let Terms\(_i\) = Terms(\text{Start}_i, \text{Pointers}_i) (1 ≤ i ≤ n). Let \( E \subseteq \text{Terms} \times \text{Terms} \) be an equality relation on terms. The forest equality closure of \( E \) with respect to \( \varphi \), denoted \( \text{Closure}^= (\varphi, E) \subseteq \text{Terms} \times \text{Terms} \) is the smallest congruence relation that satisfies the following.

- \( E \subseteq \text{Closure}^= (\varphi, E) \).
- For every \( t_i \in \text{Terms}_i \) and \( t_j \in \text{Terms}_j \) such that \( t_i \neq t_j \), we have
  \[
  (t_i, t_j) \in \text{Closure}^= (\varphi, E) \implies \{(t_i, \text{stop}_i), (t_j, \text{stop}_j)\} \subseteq \text{Closure}^= (\varphi, E)
  \]

### 3.3 Caching Programs

In this part, we will define the notion of caching programs with respect to a given reach specification \( \varphi \). To do this, we first need the notion of caching executions. Intuitively, these are executions that do not 'recompute' terms in the sense of terms computed defined above.
Definition 4. A complete or partial execution $\sigma$ is defined to be a caching execution if it satisfies the following two conditions:

1. Let $\rho$ be a prefix of the form $\rho' \cdot \text{"assume}(u = v)"$ and $t = \text{Comp}(\rho, x)$ or $\rho' \cdot \text{"}\cdot \text{\footnotesize f}(y)\text{"}$. If there is a term $t' \in \text{Terms}(\rho')$ such that $t \equiv_{\text{Closure}^*(\varphi, a(\rho'))} t'$, then it must be the case that there is some variable $z \in V$ such that $\text{Comp}(\rho', z) \equiv_{\text{Closure}^*(\varphi, a(\rho'))} t$.

Note that this condition is applicable to every sort-sensible combination of symbols.

2. Let $\rho$ be a prefix of the form $\rho' \cdot \text{"}\cdot \text{\footnotesize f}(u)\text{\,$\cdot$}
\text{\footnotesize f}(v)\text{\,$\cdot$}
\text{\footnotesize f}(w)\text{\,$\cdot$}\text{\footnotesize f}(x)\text{\,$\cdot$}\text{\footnotesize f}(y)\text{\,$\cdot$}\text{\footnotesize f}(z)\text{\,$\cdot$}\text{\footnotesize f}(a(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(b(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(c(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(d(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(e(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(f(\rho'))\text{\,$\cdot$}\text{\footnotesize f}(g(\rho'))$ where $u, v, w, x, y, z \in V_{\text{Data}}$ and $u = \text{Comp}(\rho', u), v = \text{Comp}(\rho', v)$. If there is a term $t \in \text{Terms}(\rho')$ such that $t$ is a superterm of $t_u$ or $t_v$ modulo $\Delta_{\text{Closure}^*(\varphi, a(\rho'))}$, then there must be some variable $w \in V_{\text{Data}}$ such that $\text{Comp}(\rho', w) \equiv_{\text{Closure}^*(\varphi, a(\rho'))} t$.

The definition of caching is inspired from the notion of coherence defined previously in [Mathur et al. 2019]. The first condition is the heart of the notion of caching executions. Informally, it demands that if we were to ‘recompute’ a term, then that term must already be cached in some variable. We illustrate the motivation for this requirement using the following example.

Example 1. Let $\pi_1$ be the following execution:

$$\pi_1 \overset{\Delta}{=} u := f(w) \cdot u := f(u) \cdot u := f(u) \cdot v := f(w) \cdot v := f(v) \cdot v := f(v) \cdot \text{\footnotesize assume}(u \neq v)$$

Note that the above execution is infeasible in any heap structure. However, in order to accurately determine the relationship “$u = v$” at the end of the execution, one needs to keep track of an unbounded amount of information. The first condition in Definition 4 ensures that $\pi_1$ is not a caching execution. This is because the each of the intermediate terms $f^i(w)$ ($1 \leq i < n$) are being recomputed in the second half of the execution.

In the second condition the notion of superterm modulo congruence is the following: a term $t_1$ is said to be a superterm of $t_2$ modulo a congruence $\equiv$ if there exist terms $t'_1, t'_2$ such that $t_1 \equiv t'_1, t'_2$ is a superterm of $t'_2$, and $t_2 \equiv t'_2$.

Definition 5. A program is said to be caching with respect to $\varphi$ if all its executions are caching with respect to $\varphi$.

3.4 Discussion

In this section, we discuss some of the challenges involved in extending the notion of uninterpreted programs, and the decidability and complexity-theoretic properties they entail, to handle updatable maps.

Functions and updatable maps cannot be handled uniformly; in particular, the work of [Mathur et al. 2019] does not immediately lend itself to handling updatable maps. Let us illustrate this using the following example.

Example 2. Consider the liveness property $\varphi = \{(x, y) \cdot \text{\footnotesize next} \cdot \text{\footnotesize NIL}\}$ and the straight-line program (also execution) $\pi_2 = \pi'_2 \cdot \text{\footnotesize assume}(z_2 = z_3)$ where

$$\pi'_2 \overset{\Delta}{=} \text{\footnotesize assume}(x \neq \text{\footnotesize NIL}) \cdot \text{\footnotesize assume}(y \neq \text{\footnotesize NIL}) \cdot z_1 := x \cdot \text{\footnotesize next} \cdot \text{\footnotesize assume}(z_1 \neq z_2) \cdot y \cdot \text{\footnotesize next} := z_2 \cdot z_3 := x \cdot \text{\footnotesize next}$$

The execution $\pi_2$ is feasible over general heap structures and is caching since all terms ever computed are stored in some variable. The set of heap structures in which the prefix $\pi'_2$ is feasible either interpret $x$ and $y$ to the same location or different ones. It is the final assume that rules out the latter class of models. It is also possible to indirectly imply disequalities, as can be seen in the example below using a different caching execution.
Example 3.

\[ \pi_3 \triangleq \text{assume}(x \neq \text{NIL}) \cdot \text{assume}(y \neq \text{NIL}) \cdot y \cdot \text{next} := z_1 \cdot z_2 := x \cdot \text{next} \cdot \text{assume}(z_1 \neq z_2) \]

In general this is hard to keep track in a streaming setting. In fact, this problem of aliasing can have downstream effects on the data sort which makes the problem even more complex. We illustrate this using the following example which is also caching.

Example 4. Consider the reachability specification \( \varphi = \{(x, y), \{\text{next}\}, \{\text{NIL}\}\} \) with the following execution

\[ \pi_3' \triangleq \text{assume}(x \neq \text{NIL}) \cdot \text{assume}(y \neq \text{NIL}) \cdot z_1 := x \cdot \text{next} \cdot \text{assume}(z_1 \neq z_2) \cdot y \cdot \text{next} := z_2 \]

\[ \cdot z_3 := x \cdot \text{next} \cdot k_1 := x \cdot \text{key} \cdot k_1 := f(k_1) \cdots k_1 := f(k_1) \cdot k_2 := y \cdot \text{key} \cdot k_2 := f(k_2) \cdots k_2 := f(k_2) \]

\[ \cdot \text{assume}(z_2 = z_3) \cdot \text{assume}(k_1 = k_2) \]

The above execution is feasible, but would require an unbounded amount of memory to reason as such. In fact, using a reduction similar to [Mathur et al. 2019] we show that the problem is undecidable.

Theorem 3. Given a heap-manipulating program \( P \) that is caching, the problem of checking whether there is an execution of \( P \) that is feasible on some heap structure is undecidable.

This is quite different from the result of [Mathur et al. 2019] because the corresponding problem is decidable and is in \( \text{PSPACE} \). The following result is an immediate corollary of Theorem 3.

Theorem 4. Given a heap-manipulating program \( P \) that is caching and a reachability specification \( \varphi \), the problem of checking whether it is memory safe with respect to \( \varphi \) is undecidable.

Our notion of forest data structures handles the aliasing problem while still being able to express many practical reachability specifications. In fact, with respect to forest data structures the above pathological execution \( \pi_3 \) is infeasible. The problem of memory safety is then decidable and is \( \text{PSPACE} \)-complete, as we shall discuss in the following section.

4 STREAMING CONGRUENCE CLOSURE FOR FOREST DATASTRUCTURES

In this section, we describe our algorithm for verifying memory safety of coherent programs against a given reachability specification over forest-like structures.

Recall that we restrict our attention to checking memory safety only over forest-like structures and our algorithm crucially relies properties of such structures.

Our algorithm is automata theoretic—we construct a finite state automaton such that accepts all coherent executions that are memory safe and rejects all coherent executions that are not memory safe.

Recall that our reachability specification is an indexed set of tuples \( \varphi = \{\varphi_k\}_{k=1}^n \), where \( \varphi_k = (\text{Start}_k, \text{Pointers}_k, \text{Stop}_k) \). To simplify presentation, we assume that the set of variables \( V \) in our programs is such that for every constant \( c \) appearing in the reachability specification \( \varphi \), there is a variable \( v_c \) corresponding to \( c \). Further, we assume that these variables are never over-written. We will, therefore, often interchangeably refer to these constants in \( \varphi \) by their corresponding variables, and vice versa. These assumptions can be relaxed with a more involved construction.

The automaton is a tuple \( A_{\text{MS}} = (Q, q_0, \delta) \), where \( Q \) is the set of states, \( q_0 \) is the initial state and \( \delta \) is the transition relation. Recall that executions are strings over \( \Pi \), which is also the alphabet of the automaton \( A_{\text{MS}} \). We describe each of these components below.

States. The automaton has two distinguished states \( q_{\text{infeasible}} \) and \( q_{\text{unsafe}} \). All other states are tuples of the form \((\equiv, d, P, Y, M, N, A, X)\), where each component is as follows.
• $\equiv$ is an equivalence relation over $V$ that respects sorts. We will use $[x]_{\equiv}$ to denote the set 
\{ $y$ $|$ $(x, y) \in \equiv$ \}
• $d$ is a symmetric set of pairs of the form $(c_1, c_2)$, where $c_1, c_2 \in V/\equiv$ are equivalence classes.
• $P$ gives partial mappings to functions in $\mathcal{T}_{Data}$ and pointers in $\text{FldsValLoc} \cup \text{FldsValData}$. More formally, for every $f \in \mathcal{T}_{Data}$ of arity $r$, and classes $c_1, c_2, \ldots, c_r \in V_{Data}/\equiv$, we have $P(f)(c_1, \ldots, c_r) \in V_{Data}/\equiv$ (if defined). Similarly, for every $p \in \text{FldsValLoc}$, and every $c \in V_{Loc}/\equiv$, $P(p)(c) \in V_{Loc}/\equiv$, and for every $d \in \text{FldsValData}$, and every $c \in V_{Data}/\equiv$, $P(d)(c) \in V_{Data}/\equiv$. We will denote that a map is not defined on a certain element by saying that it evaluates to undef.
• $Y = \{Y_k\}_{k=1}^n$ and $M = \{M_k\}_{k=1}^n$ are such that for every $1 \leq i \leq n$, $Y_i, M_i \subseteq V_{Loc}/\equiv$ are sets of equivalence classes over location variables.
• The sets $N, A$ and $X$ are sets of equivalence classes of location variables, i.e., $N, A, X \subseteq V_{Loc}/\equiv$.

**Initial State.** The initial state $q_0$ is the tuple $(\equiv_0, d_0, P_0, N_0, A_0, X_0)$ such that
• $\equiv_0$ is the identity relation on the set $V$ of variables,
• $P_0$ is such that for all functions and pointer fields $f$, the range of $P(f)$ is empty,
• each of $d_0, N_0, A_0$ and $(Y_0), \ldots, (Y_n)$ are $\varnothing$,
• for each $1 \leq i \leq n$, $(M_i) = \{ [c]_{\equiv_0} \mid c \in \bigcup_{i=1}^n \text{Start}_i \}$,
• $N_0 = \{ [c]_{\equiv_0} \mid c \in \bigcup_{i=1}^n \text{Stop}_i \}$,
• $X_0 = \{ [v]_{\equiv_0} \mid c \in V_{Loc} \setminus \bigcup_{i=1}^n N_0 \cup (M_i) \}$.

**Transitions.** The states $q_{\text{infeasible}}$ and $q_{\text{unsafe}}$ are absorbing states. That is, for every $s \in \Pi$, $\delta(q_{\text{infeasible}}, a) = q_{\text{infeasible}}$ and $\delta(q_{\text{unsafe}}, a) = q_{\text{unsafe}}$. In the following, we describe the transition function for every other state in $Q$. Let $q \in Q \setminus \{q_{\text{infeasible}}, q_{\text{unsafe}}\}$, $s \in \Pi$ and let $q' = \delta(q, s)$. Then, $q'$ is $q_{\text{infeasible}}$ or $q_{\text{unsafe}}$ or is of the form $q' = (\equiv', d', P', Y', M', N', A', X')$. We note here that when we describe the change on the $\equiv$ and $d$ components by adding or removing pairs we intend the closure that would preserve the component being an equivalence (respectively symmetric) relation.

1. **Case** $s = "u := v\”, u, v \in V$.
   In this case, we add the variable $u$ into the class of $v$ and appropriately update each of the components. That is, $\equiv' = (\equiv \setminus \{(u, u') \mid u \neq u', u' \in [u]_{\equiv}\}) \cup \{(u, v') \mid v' \in [v]_{\equiv}\}$. The other components of the state are the same as in $q$.

2. **Case** $s = "x := y p\”, x, y \in V_{Loc}$ and $p \in \text{FldsLoc}$.
   In this case, we need to check if the variable $y$ corresponds to a location that can be dereferenced. If not, we have a memory safety violation; otherwise, we establish the relationship $p(x) = y$ in the next state. Formally, if there is no $i$ such that $[y]_{\equiv} \in Y_i$ and if $[y]_{\equiv} \notin A$, then $q' = q_{\text{unsafe}}$. Otherwise we have that $[y]_{\equiv} \in A$ or there is a $k$ such that $[y]_{\equiv} \in Y_k$. In this case we define the tuple $q' = (\equiv', d', P', Y', M', N', A', X')$ below. Here, we need to consider the following cases.
   • Case $P(p)[[y]_{\equiv}]$ is defined and equals $[z]_{\equiv}$. In this case, $q'$ is defined in the same manner as if $s = "x := z\”$.
   • Case $P(p)[[y]_{\equiv}] = \text{undef}$. Here, for caching programs it must be the case that $[y]_{\equiv} \in Y_k$ for some $k$. Here, we create a new singleton equivalence class containing $x$ and set the

---

3When a component is described as remaining the same (modulo the equivalence relations/classes), we mean that the update to the component is as follows: if a class/tuple of classes belong to the component, then the corresponding updated equivalence classes (resp. tuple of classes) belongs to the updated component.
value of the p map, on y to be this new class. We also assert that [x] is not equal to any class in any of the Y_i's or in A. That is,
- \( \equiv' = \equiv \setminus \{(x, u) \mid u \neq x\} \cup \{(x, x)\} \).
- \( d' = \{([[u]_\equiv, [v]_\equiv) \mid u \neq x, v \neq x, ([u]_\equiv, [v]_\equiv) \in d\} \cup \{(x]_\equiv, c) \mid c \in A \cup \bigcup_{i=1}^{n} Y_i\} \).
- \( P'(p)([y]_\equiv) = [x]_\equiv. \)

For all other combinations of functions/pointers and arguments, \( P' \) behaves same as \( P \).

- The sets \( M_k \) and \( X \) are updated depending upon the pointer \( p \). If \( p \in \text{Pointers}_k \), then \( M'_k = \{[z]_\equiv \mid z \neq x, [z]_\equiv \in M_k\} \cup \{[x]_\equiv\}. \)
- Otherwise, \( X = \{[z]_\equiv \mid z \neq x, [z]_\equiv \in X\} \cup \{[x]_\equiv\}. \)
- All other components are the same as in \( q \).

3. Case \( s = "a := y \cdot d" \), \( a \in V_{\text{Data}}, y \in V_{\text{Loc}} \) and \( d \in \text{Flds}_{\text{Data}} \).

As in the previous case, \( q' = q_{\text{unsafe}} \) if \([y]_\equiv \notin \bigcup_{i=1}^{n} Y_i \cup A\). Otherwise, similar to the previous case, we have two cases to consider. As before, if there is a variable \( b \in V_{\text{Data}} \) such that \( P(d)([y]_\equiv) = [b]_\equiv \), then we treat this case as that of "\( a := b \)." Otherwise, the new equivalence relation \( \equiv' \) is such that \( \equiv' = \equiv \setminus \{(a, u) \mid u \neq a\} \cup \{(a, a)\} \), while the other components are the same as in \( q \).

4. Case \( s = "y \cdot h := u" \), \( y \in V_{\text{Loc}} \).

We uniformly handle the case of \( h \) being either a pointer field (\( \text{Flds}_{\text{Loc}} \)) or a data field (\( \text{Flds}_{\text{Data}} \)). Here we have \( q' = q_{\text{unsafe}} \) if \([y]_\equiv \notin \bigcup_{i=1}^{n} Y_i \cup A\). Otherwise, we simply change \( P \) as follows (while keeping other components same as in \( q \)):
- \( P'(f) = P(f) \) for \( f \neq h \).
- \( P'(h)([y]_\equiv) = [u]_\equiv \).
- Otherwise, for \( z \in V_{\text{Loc}} \) such that \( z \notin [y]_\equiv \), \( P'(h)([z]_\equiv) = [w]_\equiv \) if \( P(h)([z]_\equiv) = [w]_\equiv \) for some \( w \) (location or data variable depending on whether \( h \) is a pointer or data field).

5. Case \( s = "f(c_1, \ldots, c_r)" \), \( y \in V_{\text{Data}} \).

Here, we have two cases to consider again. If there is a variable \( b \in V_{\text{Data}} \) such that \( P(f)([c_1]_\equiv, \ldots, [c_r]_\equiv) = [b]_\equiv \), then we treat this case as that of "\( a := b \)." Otherwise, we add a singleton equivalence class containing \( a \) and update \( P(f) \), while keeping all other components the same (modulo the new equivalence relation).

Formally,
- \( \equiv' = \equiv \setminus \{(a, u) \mid u \neq a\} \cup \{(a, a)\} \).
- \( P'(h) \) is same as in \( q \) if \( h \neq f \). The evaluation of \( P' \) on \( f \) is described as follows.

\[
P'(f)([u_1]_\equiv, \ldots, [u_r]_\equiv) = \begin{cases} 
[a]_\equiv & \text{if for every } 1 \leq i \leq r, u_i = c_i \text{ and } a = u_i \\
[u]_\equiv & \text{otherwise if } a \notin \{u, u_1, \ldots, u_r\} \text{ and } [u]_\equiv = P(f)([u_1]_\equiv, \ldots, [u_r]_\equiv) \\
\text{undef} & \text{otherwise}
\end{cases}
\]

- All other components are the same as in \( q \).

6. Case \( s = \"\text{alloc}(x)\" \).

In this case, we create a new singleton class containing \( x \), and add this class to \( A \). We also assert that this new class is not equal to any other class. Formally,
- \( \equiv' = \equiv \setminus \{(x, u) \mid u \neq x\} \cup \{(x, x)\} \).
- \( d' = \{(\langle u_1, [u_2]_\equiv \rangle) \mid u_1 \neq u_2, ([u_1]_\equiv, [u_2]_\equiv) \in d \text{ or } u_1 = x \lor u_2 = x\} \).
- \( A' = \{[u]_\equiv \mid [u]_\equiv \in A\} \cup \{[x]_\equiv\} \).
- All other components are updated as usual.

(7) Case $s = \text{"free(x)".}$

In this case, if $[x]_\equiv \notin A \cup \bigcup_{i=1}^{n} Y_i$, then $q' = q_{\text{unsafe}}$. Otherwise, we remove the class $[x]_\equiv$ from its place in $A \cup \bigcup_{i=1}^{n} Y_i$ and add it to the set $N$. That is,

- $\equiv' = \equiv$
- $N' = \{[z]_{\equiv'} \mid [z]_{\equiv} \in N \} \cup \{[x]_{\equiv'}\}$
- $Y_i' = \{[z]_{\equiv'} \mid [z]_{\equiv} \neq [x]_{\equiv}, [z]_{\equiv} \in Y_i\}$ for every $i$
- $A = \{[z]_{\equiv'} \mid [z]_{\equiv} \neq [x]_{\equiv}, [z]_{\equiv} \in A\}$
- Other components remain the same.

(8) Case $s = \text{"assume(x = y)"}, x, y \in V_{\text{loc}}$. In this case, if $[x]_\equiv = [y]_\equiv$ then $q' = q$. Otherwise we have several cases to consider.

In each of these cases, we construct a new tuple $q'' = (\equiv'', d'', P'', Y'', N'', M'', A'', X'')$. Finally, we set $q' = q''$ if $d'' \cap \equiv'' = \emptyset$; otherwise we have $q' = q_{\text{infeasible}}$.

- The first case to consider is when (without loss of generality) $[x]_\equiv \in X \cup A \cup N \cup \bigcup_{i=1}^{n} Y_i$. In this case, we merge $[x]_\equiv$ and $[y]_\equiv$. More formally, $\equiv''$ is the smallest equivalence relation such that $\equiv \cup \{(x, y)\} \subseteq \equiv''$. Further for every $i$ and for every $z \in V_{\text{loc}}$ such that $z \notin [x]_\equiv$, $[z]_{\equiv''} \in Y_i''$ iff $[z]_{\equiv} \in Y_i$ (similarly for the sets $M_i, X, A, N$). The other components of $q''$ are the same as in $q$ (modulo the new equivalence classes).
- Otherwise, consider the case when $[x]_\equiv \in M_i$ for some $i$. In this case, in addition to adding $(x, y)$ we also add the pair $(x, \text{stop}_i)$. Similarly if $[y]_\equiv \in M_j$ for some $j$ we add $(y, \text{stop}_j)$. Construct the state $q''$ with $\equiv''$ being the smallest equivalence relation including these new pairs (and other components remaining the same).

(9) Case $s = \text{"assume(x \neq y)"}, x, y \in V_{\text{loc}}$. Similarly as above, in this case when $([x]_\equiv, [y]_\equiv) \in d$ we have $q' = q$. If $[x]_\equiv = [y]_\equiv$ then $q' = q_{\text{infeasible}}$. Otherwise, we have the following cases:

- $[x]_\equiv = \text{stop}_i$ and $[y]_\equiv \in M_i$ for some $i$. In this case, we simply put the equivalence class of $y$ into $Y_i$ and assert that $[y]_\equiv$ is unequal to all other classes. More formally:
  - $\equiv'' = \equiv$
  - $Y_i' = Y_i \cup \{[y]_\equiv\}$
  - $d' = d \cup \{([y]_\equiv, [z]_\equiv) \mid z \notin [y]_\equiv\}$
  - The other components remain the same.
- Otherwise, we simply update $d' = d \cup \{([x]_\equiv, [y]_\equiv)\}$ and all other components remain the same.

(10) Case $s = \text{"assume(a = b)"}, a, b \in V_{\text{data}}$. In this case, we merge equivalence classes repeatedly and perform a ‘local congruence closure’. We shall construct a ‘state’ $q''$ to determine if the transition must be to $q_{\text{infeasible}}$. More formally, we shall define the state $q''$ with the $\equiv''$ component as the smallest equivalence relation such that: (a) $\equiv \subseteq (a, b) \subseteq \equiv''$ (b) If $(u_i, v_i) \in \equiv$ for $1 \leq i \leq r$ and $[w]_{\equiv'} = f([u_1]_{\equiv'}, \ldots, [u_r]_{\equiv'})$, $[w']_{\equiv'} = f([v_1]_{\equiv'}, \ldots, [v_r]_{\equiv'})$ then $(w, w') \in \equiv''$.

The other components remain the same. In particular, it is correct to retain the $P$ component since the above construction is a congruence relation. Finally, if there exist $u, v \in V_{\text{data}}$ such that $(u, v) \in \equiv''$ and $([u]_{\equiv'}, [v]_{\equiv'}) \in d''$ then $q' = q_{\text{infeasible}}$. Otherwise, $q' = q''$.

(11) Case $s = \text{"assume(a \neq b)"}, a, b \in V_{\text{data}}$. Similarly as above, if $[a]_\equiv = [b]_\equiv$ then $q' = q_{\text{infeasible}}$.

Otherwise, we update $d' = d \cup \{([a]_\equiv, [b]_\equiv)\}$ and all other components remain the same.

The following theorem states the correctness of the automaton $A_{MS}$. 

Theorem 5. Let $\sigma$ be a caching execution and let $\varphi$ be a reachability specification and let $q$ be the state of the automaton $A_{MS}$ after reading $\sigma$. Then, $q = q_{\text{unsafe}}$ iff there is a forest datastructure $M$ (with respect to $\varphi$) such that $\sigma$ violates memory safety on $M$.

The problem of checking if a caching program is memory safe against a given specification is decided as follows. Recall that the set of executions of a given program $P$ constitutes a regular language $\text{Exec}(P)$. Let $L(A_{MS})$ denote the set of executions $\sigma \in \Pi^*$ that go to the state $q_{\text{unsafe}}$. Then, the problem of checking if $P$ is memory safe reduces to checking if the intersection $\text{Exec}(P) \cap L(A_{MS})$ is empty. This gives us the following result.

Theorem 6. The memory safety verification problem over forest datastructures for caching programs is decidable and is PSPACE-complete.

Next, we show that the problem of checking caching is also decidable. To address the problem of checking caching, we construct an automaton $A_{\text{caching}}$ similar to $A_{MS}$ that keeps track of the following information. For every function/pointer $f$ of arity $r$, and for every tuple $(x_1, \ldots, x_r)$ of variables (of appropriate sorts), each state of the automaton $A_{\text{caching}}$ maintains a boolean predicate denoting whether or not $f(x_1, \ldots, x_r)$ has been computed in any execution that reaches the state. This gives us our next result.

Theorem 7. The problem of checking whether a given program is caching with respect to a given reach specification is decidable in PSPACE.

5 IMPLEMENTATION AND EVALUATION

We implemented a tool for deciding memory safety of forest datastructures based on the streaming congruence closure algorithm from Section 4. The tool is ~2000 lines of Ocaml 4.07.0 code. It takes as input a program from the grammar presented in Section 2.1.1 annotated with a reachability specification, as in Section 2.1.4. The tool does not explicitly construct the automaton as the number of states is exponential in the number of program variables. Instead, the core programming modules involve implementing the state transformation function for each of the instruction symbols in our regular language for executions from Section 2.1.2.

Algorithm: At a high level, the algorithm iteratively constructs the set of states that are reachable at any control location of the program. It begins with the singleton set containing the initial state, in which all program variables are in singleton equivalence classes and there are no known disequalities nor any function mappings between classes. The algorithm proceeds by reading each program instruction sequentially, applying the state transformer pointwise to the current set of reachable states. The size of the set grows when it encounters if-then-else and while instructions (due to joins), the latter of which involves repeated processing of the loop body and conditional until the set of reachable states arrives at a fixed point. This is guaranteed to happen because the number of states is finite. If the algorithm detects a memory safety violation it halts and reports the error. In addition to memory safety, the algorithm monitors the caching property as it processes the input. It keeps track of all equivalence classes that are dropped by remembering which functions or pointers were used to construct the class and to which classes correspond the inputs. Whenever new terms are computed, the algorithm asserts that the term has not yet been computed using preexisting classes. If not, the algorithm reports a failure of caching and halts.

We note that the algorithm is actually a bit more general than we have described thus far. Once the computation has processed the last instruction in the program, the set of final states can be inspected for feasibility. Any assertion in the form of a boolean combination of equality statements on program variables can be checked. This can be accomplished by appending the negated assertion to the end of the program and then checking that all reachable states are infeasible.
Benchmarks: We are not aware of any existing decision procedures (sound and complete) for memory safety. As far as we are aware, our tool is the first decision procedure that can handle the benchmarks that we propose.

In evaluating the tool, we seek to answer the following basic questions about caching programs and our algorithm. First, is it the case that the most natural way to write pointer-manipulating one-pass programs on lists and trees results in caching? Second, for caching programs with and without memory safety violations, is the algorithm able to verify the memory safe programs and find violations in the ones that are not? And how fast is the algorithm? Note that since we do abstract the primitive types and functions/relations on them, it is not clear that the tool will be able to prove the memory safe programs correct.

To answer the first question, we wrote natural pointer-manipulating programs over singly linked lists (lists, sorted lists) and tree data structures (bst, avl, rotations of trees, etc.) in our input language, and evaluated the tool on them to find if they are caching. To answer the second question, we evaluated the tool for memory safety checking on these benchmarks.

The first column of Table 1 gives the set of programs in our benchmark. These are typically single pass algorithms over an input data structure. For example, finding a key in a binary search tree or reversing a linked list are single pass algorithms.

The names of the programs indicate whether or not the program truly contains an unsafe memory access (i.e., the ground truth). Programs whose names end in `unsafe` were obtained by introducing one of two possible memory safety errors into their `safe` counterparts. The first kind of memory unsafe program we test is reference to unallocated memory locations. That is, any program that attempts to read or write to a location that is unallocated. The second kind of unsafe program involves freeing unallocated memory locations.

One example of the first kind is illustrated in `sll-copy-all`, which copies the contents of a linked list into a freshly allocated list. In this example, the program steps through the input list in a while loop until it reaches `NIL`. In each iteration, a new node is allocated, initialized with the contents of the current node, and connected to the end of the new list. The program relies on the invariant that the new list has a next node to step to whenever the old list does. Thus, it does not perform a `NIL` check when advancing along the next pointer for the new list. The `sll-copy-all-unsafe` fails to maintain the invariant by incorrectly adding the freshly allocated node to the new list. An example for errors of the second kind (freeing memory locations that may not be allocated) can be found in `sll-delete-between-unsafe`. In this example, the task is to delete all nodes in a linked list that have key values in a certain range. The mistake in this example happens when the program has found a node to delete but, instead of saving the next node and deleting the current node, it instead frees the next node, which may be unallocated.

Discussion of results: Table 1 shows the result of our tool, when run on a machine running Ubuntu 18.04 with an Intel i7 processor at 2.6 GHz. Columns 3-6 pertain to the operation of the algorithm on the benchmarks. Column 3 gives whether or not the benchmark fails the caching condition. Our tool was able to terminate and identify whether the programs are memory safe accurately on all caching programs. Column 4 depicts whether or not an unsafe memory access was detected. Column 5 gives the total number of states that are reachable at the end of the program. Note that noncaching and memory unsafety preclude each other in the table. Upon detecting either, the algorithm halts. (and we do not report the number of reachable states). Column 6 gives the total running time of the tool on each benchmark, which is negligible in all cases. Note that the number of reachable states for each example is also quite small relative to the total number of possible states, which grows faster than the Bell numbers. That our algorithm only examines a small fraction of
### Table 1. Evaluation

<table>
<thead>
<tr>
<th>Program</th>
<th>LOC</th>
<th>Noncontiguous</th>
<th>Safe</th>
<th># States</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sll-append-safe</td>
<td>19</td>
<td>no</td>
<td>✓</td>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-append-unsafe</td>
<td>20</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-copy-all-safe</td>
<td>27</td>
<td>no</td>
<td>✓</td>
<td>6</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-copy-all-unsafe</td>
<td>29</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-delete-all-safe</td>
<td>56</td>
<td>no</td>
<td>✓</td>
<td>58</td>
<td>0.016</td>
</tr>
<tr>
<td>sll-delete-all-unsafe</td>
<td>58</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.013</td>
</tr>
<tr>
<td>sll-deletebetween-safe</td>
<td>42</td>
<td>no</td>
<td>✓</td>
<td>53</td>
<td>0.014</td>
</tr>
<tr>
<td>sll-deletebetween-unsafe</td>
<td>44</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-find-safe</td>
<td>16</td>
<td>no</td>
<td>✓</td>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-find-unsafe</td>
<td>18</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-back-safe</td>
<td>20</td>
<td>no</td>
<td>✓</td>
<td>3</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-back-unsafe</td>
<td>20</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-front-safe</td>
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<td>✓</td>
<td>1</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-front-unsafe</td>
<td>9</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-safe</td>
<td>50</td>
<td>no</td>
<td>✓</td>
<td>12</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-insert-unsafe</td>
<td>50</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-reverse-safe</td>
<td>12</td>
<td>no</td>
<td>✓</td>
<td>3</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-reverse-unsafe</td>
<td>12</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-concat-safe</td>
<td>17</td>
<td>no</td>
<td>✓</td>
<td>4</td>
<td>0.013</td>
</tr>
<tr>
<td>sll-sorted-concat-unsafe</td>
<td>17</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-insert-safe</td>
<td>50</td>
<td>no</td>
<td>✓</td>
<td>12</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-insert-unsafe</td>
<td>50</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-merge-noncontiguous</td>
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<td>yes</td>
<td>—</td>
<td>—</td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-merge-safe</td>
<td>74</td>
<td>no</td>
<td>✓</td>
<td>62</td>
<td>0.015</td>
</tr>
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<td>sll-sorted-merge-unsafe-1</td>
<td>69</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>sll-sorted-merge-unsafe-2</td>
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<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>bst-find-safe</td>
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<td>✓</td>
<td>21</td>
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</tr>
<tr>
<td>bst-find-unsafe</td>
<td>25</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>bst-insert-safe</td>
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<td>no</td>
<td>✓</td>
<td>29</td>
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<td>bst-insert-unsafe</td>
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<td>✗</td>
<td></td>
<td>0.013</td>
</tr>
<tr>
<td>bst-remove-root-safe</td>
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<td>no</td>
<td>✓</td>
<td>12</td>
<td>0.012</td>
</tr>
<tr>
<td>bst-remove-root-unsafe</td>
<td>54</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>avl-balance-safe</td>
<td>190</td>
<td>no</td>
<td>✓</td>
<td>48</td>
<td>0.018</td>
</tr>
<tr>
<td>avl-balance-unsafe</td>
<td>111</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>tree-rotate-left-safe</td>
<td>25</td>
<td>no</td>
<td>✓</td>
<td>3</td>
<td>0.012</td>
</tr>
<tr>
<td>tree-rotate-left-unsafe</td>
<td>20</td>
<td>no</td>
<td>✗</td>
<td></td>
<td>0.012</td>
</tr>
</tbody>
</table>

the total state space is encouraging, and suggests that it may scale well for much larger and more complex programs.

### 6 CONCLUSIONS AND FUTURE WORK

We presented a class of programs, called caching programs, working on forest structures, for which memory safety is decidable. We also proved membership of programs in this class is decidable. We...
showed through a prototype implementation of our tool and a set of benchmarks that single-pass algorithms on forest structures typically fall in our decidable class, and for them we can verify memory safety accurately despite treating functions and relations on primitive type domains as uninterpreted.

The most compelling future direction is to adapt the technique in this paper to provide a memory safety analysis tool for a standard programming language (such as C/C++), handling the rest of the programming language using abstractions (e.g., arrays, allocation of varying blocks of memory, etc.). We believe that our automata-based algorithm will scale well. Realizing the techniques presented herein in a full-fledged memory safety analysis tool would be interesting.

On the theoretical front, an interesting problem is to generalize our results beyond forest structures. As argued in the introduction, this seems challenging. In fact, preliminary investigation suggests that the complexity of verifying memory safety for non-forest structures is likely to be exponentially more expensive. Nevertheless, finding a class of programs (similar to caching programs) over any datastructure for which memory safety is decidable is an interesting open problem. Finally, a sound, incomplete but effective technique to prove/disprove memory safety of programs that make multiple passes on a datastructure is an interesting direction that deserves exploration.

REFERENCES


