Computing Information Flow Using Symbolic Model-Checking

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Outline

Introduction

- Quantifying information leakage - Inferring information about inputs by observing public outputs

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- No leakage $\implies$ Outputs independent of inputs
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- Comparing leakage across programs - less leakage is desirable

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2. Shannon entropy leakage measures expected number of guesses required to correctly guess the secret input

$$\mathrm{SE}_{\mathsf{U}}(P) = \log |S| - \frac{1}{|S|} \sum_{o \in O} |P^{-1}(o)| \log |P^{-1}(o)|$$

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Dining Cryptographers

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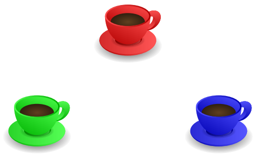

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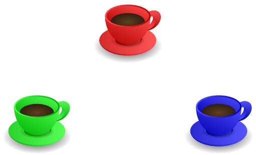

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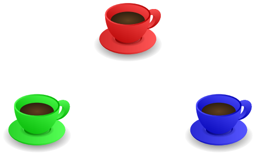

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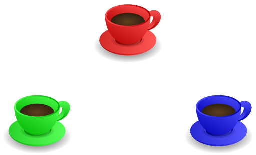

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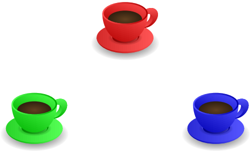

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- Determine if the NSA paid or not w/o revealing information about cryptographers

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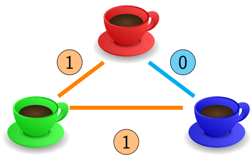

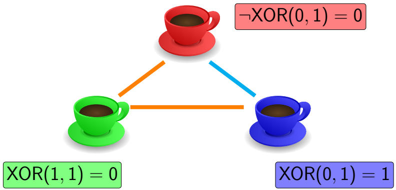

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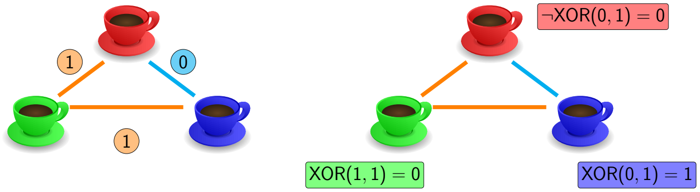

Stage-1 (left) and Stage-2 (right)

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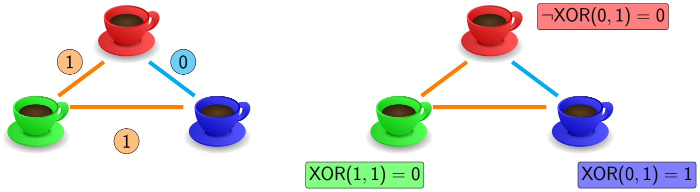

Stage-1 (left) and Stage-2 (right)

$$\text{XOR}(\text{Announcement}_A, \text{Announcement}_B, \text{Announcement}_C) = 0$$

iff

NSA paid for the dinner

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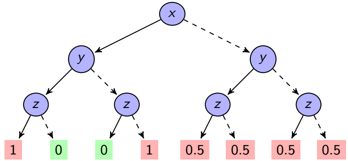

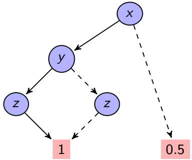

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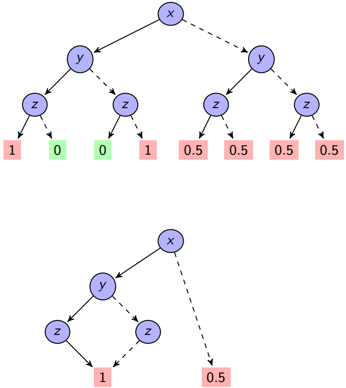

ADD (up) and its reduced form (bottom)

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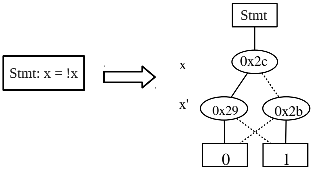

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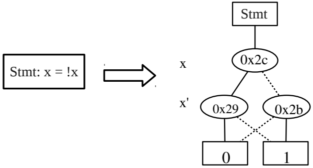

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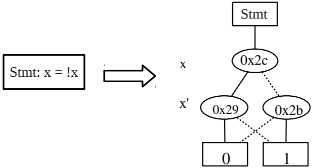

- Compose statements
- Arrive at a fixed point (Summary μ)

Min Entropy : Symbolic Algorithm

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Algorithm 6: Symbolic computation of min-entropy leakage of a probabilistic program

Input: $\mathcal{G}, \mathcal{G}'$ and T_P the summary of P.

Output: $\mathrm{ME}_{\mathrm{U}}(P)$

begin

  $T_{\mathsf{out},P} \longleftarrow \mathsf{abstract}(\max, \mathcal{G}, T_P)$

  $\mathsf{sum}_{\mathsf{out}} \longleftarrow \mathsf{val}(\mathsf{abstract}(+, \mathcal{G}', T_{\mathsf{out},P}))$

  $T_{\mathsf{term},P} \longleftarrow \mathsf{abstract}(+, \mathcal{G}', T_P)$

  $\mathsf{sum}_{\mathsf{out}} \longleftarrow \mathsf{sum}_{\mathsf{out}} + (1 - \mathsf{val}(\mathsf{abstract}(\min, \mathcal{G}, T_{\mathsf{term},P})));$

  return $\log sum_{out}$

Shannon Entropy : Symbolic Algorithm

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Algorithm 8: Symbolic computation of Shannon entropy leakage of a probabilistic program

Input: $\mathcal{G}, \mathcal{G}'$ and T_P the summary of P.
Output: $\mathrm{SE}_{\mathrm{U}}(P)$

Let n be the number of variables in $\mathcal{G}$.

begin

- $T_{\text{norm-eq-size},P} \longleftarrow \text{divide}(\text{abstract}(+, \mathcal{G}, T_P), 2^n)$
- $\text{val}_{\text{out}} \longleftarrow (\text{- val}(\text{abstract}(\star, \mathcal{G}', T_{\text{norm-eq-size},P})))$
- $T_{\text{term},P} \longleftarrow \text{abstract}(+, \mathcal{G}', T_P)$
- $\text{prob}_{\text{out,non-term}} \longleftarrow (1 - \frac{\text{val}(\text{abstract}(+, \mathcal{G}, T_{\text{term},P}))}{2^n})$
- $\text{val}_{\text{out,non-term}} \longleftarrow (\text{- prob}_{\text{out,non-term}} \log \text{prob}_{\text{out,non-term}})$
- $T_{\text{norm-}\star\text{out},P} \longleftarrow \text{divide}(\text{abstract}(\star, \mathcal{G}', T_P), 2^n)$
- $\text{val}_{\text{cond}} \longleftarrow (\text{-val}(\text{abstract}(+, \mathcal{G}, T_{\star\text{out},P})))$
- $T_{\text{non-term},P} \longleftarrow \text{subtract}(1, T_{\text{term},P})$
- $\text{val}_{\text{cond,non-term}} \longleftarrow (\text{-}\frac{\text{val}(\text{abstract}(\star, \mathcal{G}, T_{\text{non-term-prob},P}))}{2^n})$
- **return** $(val_{out} + val_{out,non\text{-}term} - val_{cond} - val_{cond,non\text{-}term})$

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```
define N 32

define DEFAULT_INT_BITS N

unsigned int var1;
bool g;

module void f(unsigned int v, bool z){

    bool k;
    pchoice
    :: 0.2 -> label2: k = g && z;
    :: 0.8 -> var1 = var1 + v;
    choicep
}

module void main(){

    var1 = 53;
    pchoice
    :: 0.3 -> label1: g = true;
    :: 0.7 ->  f(var1, !g);
    choicep
}
```

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Salient features:

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- Consistently outperforms **sqifc** (Malacaria et. al)

Moped-QLeak

Example	Order	ME	SE	Time	Data types
Illustrative Example	I	3	2.03966e-05	0.215	bool
Electronic Purse	D	2	2	0.009	5 bit integers (Restricted)
Mix and Duplicate	S	16	16	0.041	bool
Binary Search	I	16	16	9.307	bool
Sanity Check	I	4	1.168e-7	0.060	bool
Implicit Flow	D	2.8074	1.757e-07	0.016	30 bit integers
Implicit Flow	D	2.8074	0.003	0.010	15 bit integers
Implicit Flow	D	2.8074	4.67189e-08	0.190	bool
Masked Copy	I	16	16	0.038	bool
Sum Query	D	4.80735	4.35132	0.034	5 bit integers (Restricted)

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Tool demonstration

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